Evolving Transport induced by a Surface Wave Propagating along a Vacuum-Matter Interface

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Abstract

We perform hydrodynamic derivations of the entrainment of matter (gas) induced by a surface elastic wave (representing possibly vacuum fluctuations) propagating along the flexible vacuum-matter interface by considering the nonlinear coupling between the interface and the rarefaction effect via a boundary perturbation method. The critical reflux values associated with the product of the second-order (unit) body forcing and the Reynolds number (representing the viscous dissipations and is the ratio of wave inertia and viscous dissipation effects) decrease as the Knudsen number (representing the rarefaction measure) increases from zero to 0.1. We obtained the critical bounds for matter-freezed or zero-volume-flow-rate states corresponding to specific Reynolds numbers and wave numbers which might be linked to the dissipative evolution of the Universe. Our results also show that for certain time-averaged evolution of the matter (gas) there might be existence of negative-pressure states.

Keywords : cosmic flows, physics of the early universe, gravity waves/theory

Contents

1 Introduction

In today's standard inflationary scenario the amplified vacuum fluctuations of the inflation field provide the seeds for large-scale structure formation [1]. It is thus of interest to understand the effect of these quantum fluctuations on the universe dynamics and the implications they may have for cosmological observations.

The mean cosmic density of dark matter (plus baryons) is now pinned down to be only ca. 30% of the so-called critical density corresponding to a 'flat'-Universe. However, other recent evidence—microwave background anisotropies, complemented by data on distant supernovae—reveals that our Universe actually is 'flat', but that its dominant ingredient (ca. 70% of the total mass energy) is something quite unexpected: 'dark energy' pervading all space, with negative pressure. We do know that this material is very dark and that it dominates the internal kinematics, clustering properties and motions of galactic systems. Dark matter is commonly associated to weakly interacting particles (WIMPs), and can be described as a fluid with vanishing pressure. It plays a crucial role in the formation and evolution of structure in the universe and it is unlikely that galaxies could have formed without its presence [1-2]. Analysis of cosmological mixed dark matter models in spatially flat Friedmann Universe with zero Λ term have been presented before. For example, we can start from the Einstein action describing the gravitational forces in the presence of the cosmological constant [3]

$$S = \frac{1}{2k_N^2}\int \sqrt{-g}\, R - \int \sqrt{-g}\, \Lambda + S_m,$$

where S_m is the contribution of the matter and radiation, $k_N^2 = 8\pi G_N = 8\pi/M_P^2$, G_N is the Newton's constant and $M_P = 1.22 \times 10^{19}\mathrm{GeV}/c^2$ is the Planck mass. The set of equations governing the evolution of the universe is completed by the Friedman equations for the scale factor $(a(t))$

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G_N}{3}(\rho + \Lambda) - \frac{k}{a^2},$$

$$(\frac{\ddot{a}}{a})^2 = -\frac{4\pi G_N}{3}(\rho + \Lambda + 3(p - \Lambda)),$$

where $k = 0$ is for the flat-Universe, ρ and p is the density and pressure [4-5]. A large majority of dark energy models describes dark energy in terms of the equation of state (EOS) $p_d = \omega \, \rho_d$, where ω is the parameter of the EOS, while p_d and ρ_d denote the pressure and the energy density of dark energy, respectively. The value $\omega = -1$ is characteristic of the cosmological constant, while the dynamical models of dark energy generally have $\omega \geq -1$.

Meanwhile, it is convenient to express the mean densities ρ_i of various quantities in the Universe in terms of their fractions relative to the critical density: $\Omega_i = \rho_i/\rho_{crit}$. The theory of cosmological inflation strongly suggests that the total density should be very close to the critical one ($\Omega_{tot} \sim 1$), and this is supported by the available data on the cosmic microwave background (CMB) radiation. The fluctuations observed in the CMB at a level ca. 10^{-5} in amplitude exhibit a peak at a partial wave $l \sim 200$, as would be produced by acoustic oscillations in a flat Universe with $\Omega_{tot} \sim 1$. At lower partial waves, $l \gg 200$, the CMB fluctuations are believed to be dominated by the Sachs-Wolfe effect due to the gravitational potential, and more acoustic oscillations are expected at $l > 200$, whose relative heights depend on the baryon density Ω_b. At even larger values, $l \geq 1000$, these oscillations should be progressively damped away [1-2,3,6].

Influential only over the largest of scales-the cosmological horizon-is the outermost species of invisible matter: the vacuum energy (also known by such names as dark energy, quintessence, x-matter, the zero-point field, and the cosmological constant Λ). If there is no exchange of energy between vacuum and matter components, the requirement of general covariance implies the time dependence of the gravitational constant G. Thus, it is interesting to look at the interacting behavior between the vacuum (energy) and the matter from the macroscopic point of view. One related issue, say, is about the dissipative matter of the flat Universe immersed in vacua [7] and the other one is the macroscopic Casimir effect with the deformed boundaries [8].

Theoretical (using the Boltzmann equation) and experimental studies of interphase nonlocal transport phenomena which appear as a result of a different type of nonequilibrium representing propagation of a surface elastic wave have been performed since late 1980s [9-10]. These are relevant to rarefied gases (RG) flowing along deformable elastic slabs with the dominated parameter being the Knudsen number (Kn = mean-free-path/L_d, mean-free-path (mfp) is the mean free path of the gas, L_d is proportional to the distance between two slabs) [11-13]. The role of the Knudsen number is similar to that of the Navier slip parameter N_s [14]; here, $N_s = \mu S/d$ is the dimensionless Navier slip parameter; S is a proportionality constant as $u_s = S\tau$, τ : the shear stress of the bulk velocity; u_s : the dimensional slip velocity; for a no-slip case, $S = 0$, but for a no-stress condition. $S = \infty$, μ is the fluid viscosity, d is one half of the distance between upper and lower slabs).

1.1 Present Approaches and Objectives

Here, the transport driven by the wavy elastic vacuum-matter interface will be presented. The flat-Universe is presumed and the corresponding matter is immersed in vacua with the interface being flat-plane like. We adopt the macroscopic or hydrodynamical approach and simplify the original system of equations (related to the momentum and mass transport) to one single

higher-order quasi-linear partial differential equation in terms of the unknown stream function. In this study, we shall assume that the Mach number $Ma \ll 1$, and the governing equations are the incompressible Navier-Stokes equations which are associated with the relaxed slip velocity boundary conditions along the interfaces [11-14]. We then introduce the perturbation technique so that we can solve the related boundary value problem approximately. To consider the originally quiescent gas for simplicity, due to the difficulty in solving a fourth-order quasi-linear complex ordinary differential equation (when the wavy boundary condition are imposed), we can finally get an analytically perturbed solution and calculate those physical quantities we have interests, like, time-averaged transport or entrainment, perturbed velocity functions, critical unit body forcing corresponding to the freezed or zero-volume-flow-rate states. These results might be closely linked to the vacuum-matter interactions (say, macroscopic Casimir effects) and the evolution of the Universe (as mentioned above : the critical density [1-2]). Our results also show that for certain time-averaged evolution of the matter (the maximum speed of the matter (gas) appears at the center-line) there might be existence of negative-pressure states.

2 Formulations

2.1 Preview of Mesoscopic Approach

The effects of elastic or deformable interfaces, like surface elastic waves (SLW) interacted with volume and surface phonons (propagating along the elastic boundaries), upon the entrained transport of gases have been studied since early 1950s [9-10], however, the mathematical difficulty is essential therein. The role of elastic macroscopic walls resembles that of microscopic phonons. As presented in [9], for the description of the transport processes in the non-equilibrium gas-solid system including the processes occurring in the case of propagation of an elastic wave in a solid, we need to solve

$$\frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial t} + \mathbf{v}\cdot\frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial \mathbf{x}} = I_g(\{f\}),$$

$$\frac{\partial n(\mathbf{x},\mathbf{k}_j,t)}{\partial t} + \mathbf{c}_j\cdot\frac{\partial n(\mathbf{x},\mathbf{k}_j,t)}{\partial \mathbf{x}} = I_v(\{n\}),$$

$$\frac{\partial H(\mathbf{r},\mathbf{K}_\xi,t)}{\partial t} + \mathbf{c}_\xi\cdot\frac{\partial H(\mathbf{r},\mathbf{K}_\xi,t)}{\partial \mathbf{r}} = I_s(\{f,n,H\}),$$

and the associated boundary conditions (scattering and interacting laws near the interface)

$$|v_r| f^+ = \int_{v_i<0} d\mathbf{v}_i |v_i| f^-(\mathbf{v}_i) W(\mathbf{v}_i \to \mathbf{v}_r),$$

$$\frac{|c_r|}{L_t} n^-(\mathbf{k}_j) = \sum_{\mathbf{k}_{j_1}(c_i>0)} [\frac{c_i}{L_t} n^+(\mathbf{k}_{j_1}) + \bar{N}_g(\mathbf{k}_{j_1}) + \bar{N}_p(\mathbf{k}_{j_1})] V_p(\mathbf{k}_{j_1} \to \mathbf{k}_j;\omega).$$

f, n, and H denote the distribution function for gases, volume phonons, and surface phonons, respectively. I_g, I_v, and I_s are the corresponding collision integrals. Please see the details in [9] for other notations or symbols.

As the details of the vacuum energy (also known by such names as dark energy, quintessence, the zero-point field, and the cosmological constant [4]) and dark matter (possible candidates : non-luminous baryonic matter in the form of large planets or dead stars or weakly interacting elementary particles that pervade large regions of space [2]) are not known so far and we only consider the overall effect of the matter, thus the detailed microscopic collision or scattering occurring along the interface and transfer of the energy from matter to the vacuum energy (which could be cosmological constant) will not be treated here and is beyond our present approach.
We noticed that particle physics candidates for dark matter fall into three basic categories depending upon their masses and interactions [2]. Weakly interacting particles that are in thermal equilibrium at a very early stage in the evolution of the universe will eventually fall out of equilibrium as the universe expands. The decoupling time (or temperature) when this occurs depends on the expansion rate of the universe as well as the couplings of these particles to other particles that are still in equilibrium. Particles that are (non-)relativistic at the time that galaxies start to form are referred to as (cold) hot dark matter. The simplest example of hot dark matter is a neutrino with a very small mass ($< O(20)$ eV) and of cold dark matter a very heavy neutrino (~ 100 GeV). In both cases the interaction rates are determined by the standard model of electroweak interactions (SM). The third type of particle dark matter can arise during the QCD phase transition as the universe cools down. In this case the result can be a gas of axions [4].)
To escape from above (many-body problems) difficulties, we plan to use the macroscopic approach which is a complicated extension of previous approaches [10] because we still take the rarefaction effect (due to the incomplete accomodations of the momenta or energy for collisions or reflections of particles from the matter-vacuum interface which are approximated and represented by the Knudsen number or the slip velocities [11-13]) into account.

2.2 Interface Treatment

We consider a two-dimensional matter-region of uniform thickness which is approximated by a homogeneous rarefied gas (Newtonian viscous fluid). The equation of motion is

$$(\lambda_L + \mu)\mathrm{grad}\,\mathrm{div}\,\mathbf{u} + \mu\nabla\mathbf{u} + \rho\mathbf{p} = \rho\frac{\partial^2\mathbf{u}}{\partial t^2}, \tag{1}$$

where λ_L and μ are Lamé constants, $\mathbf{u}$ is the displacement field (vector), ρ is the mass density and $\mathbf{p}$ is the body force for unit mass. The Navier-Stokes equations, valid for Newtonian fluids (both gases and liquids), has been a mixture of continuum fluid mechanics ever since 1845 following the seemingly definitive work of Stokes and others [15], who proposed the following rheological constitutive expression for the fluid deviatoric or viscous stress (tensor) $\mathbf{T}$: $\mathbf{T} = 2\mu\nabla\mathbf{u} + \lambda_L\mathbf{I}\nabla\cdot\mathbf{u}$.
The flat-plane boundaries of this matter-region or the vacuum-matter interfaces are rather flexible and presumed to be elastic, on which are imposed traveling sinusoidal waves of small amplitude a (possibly due to vacuum fluctuations). The vertical displacements of the upper and lower interfaces ($y = d$ and $-d$) are thus presumed to be η and $-\eta$, respectively, where

$\eta = a\cos[2\pi(x-ct)/\lambda]$, λ is the wave length, and c the wave speed. x and y are Cartesian coordinates, with x measured in the direction of wave propagation and y measured in the direction normal to the mean position of the vacuum-matter interfaces. The schematic plot of above features is shown in Fig. 1.

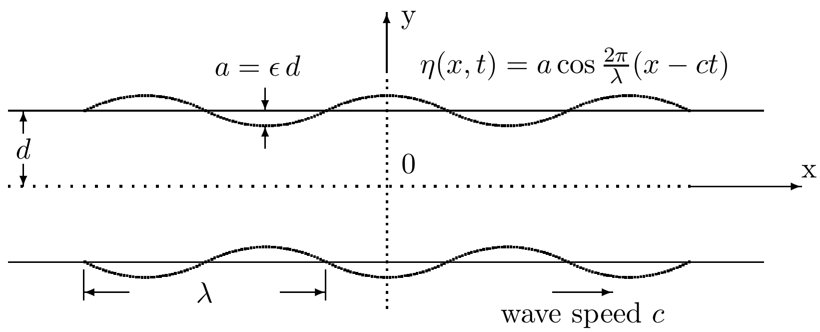

Fig. 1 Schematic diagram of the wavy motion of the vacuum-matter interfaces.

It would be expedient to simplify these equations by introducing dimensionless variables. We have a characteristic velocity c and three characteristic lengths a, λ, and h. The following variables based on c and d could thus be introduced :

$$x' = \frac{x}{d}, \quad y' = \frac{y}{d}, \qquad u' = \frac{u}{c}, \quad v' = \frac{v}{c}, \quad \eta' = \frac{\eta}{d}, \quad \psi' = \frac{\psi}{c\,d}, \quad t' = \frac{c\,t}{d}, \quad p' = \frac{p}{\rho c^2},$$

where ψ is the dimensional stream function, u and v are the velocities along the x- and y-directions; ρ is the density, p (its gradient) is related to the (unit) body forcing. The primes could be dropped in the following. The amplitude ratio ϵ, the wave number α, and the Reynolds number (ratio of wave inertia and viscous dissipation effects) Re are defined by

$$\epsilon = \frac{a}{d}, \quad \alpha = \frac{2\pi d}{\lambda}, \quad Re = \frac{c\,d}{\nu}.$$

We shall seek a solution in the form of a series in the parameter ϵ :

$$\psi = \psi_0 + \epsilon\psi_1 + \epsilon^2\psi_2 + \cdots,$$

$$\frac{\partial p}{\partial x} = (\frac{\partial p}{\partial x})_0 + \epsilon(\frac{\partial p}{\partial x})_1 + \epsilon^2(\frac{\partial p}{\partial x})_2 + \cdots,$$

with $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$. The 2D (x- and y-) momentum equations and the equation of continuity could be in terms of the stream function ψ if the p-term (the specific body force density, assumed to be conservative and hence expressed as the gradient of a time-independent potential energy function) is eliminated. The final governing equation is

$$\frac{\partial}{\partial t}\nabla^2\psi + \psi_y\nabla^2\psi_x - \psi_x\nabla^2\psi_y = \frac{1}{Re}\nabla^4\psi, \qquad \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \tag{2}$$

and subscripts indicate the partial differentiation. Thus, we have

$$\frac{\partial}{\partial t}\nabla^2\psi_0 + \psi_{0y}\nabla^2\psi_{0x} - \psi_{0x}\nabla^2\psi_{0y} = \frac{1}{Re}\nabla^4\psi_0, \tag{3}$$

$$\frac{\partial}{\partial t}\nabla^2\psi_1+\psi_{0y}\nabla^2\psi_{1x}+\psi_{1y}\nabla^2\psi_{0x}-\psi_{0x}\nabla^2\psi_{1y}-\psi_{1x}\nabla^2\psi_{0y}=\frac{1}{Re}\nabla^4\psi_1, \tag{4}$$

$$\frac{\partial}{\partial t}\nabla^2\psi_2+\psi_{0y}\nabla^2\psi_{2x}+\psi_{1y}\nabla^2\psi_{1x}+\psi_{2y}\nabla^2\psi_{0x}-$$
$$\psi_{0x}\nabla^2\psi_{2y}-\psi_{1x}\nabla^2\psi_{1y}-\psi_{2x}\nabla^2\psi_{0y}=\frac{1}{Re}\nabla^4\psi_2, \tag{5}$$

and other higher order terms. The (matter) gas is subjected to boundary conditions imposed by the symmetric motion of the vacuum-matter interfaces and the non-zero slip velocity : $u=\mp$ Kn du/dy [11-13], $v=\pm\partial\eta/\partial t$ at $y=\pm(1+\eta)$, here Kn=mfp/$(2d)$. The boundary conditions may be expanded in powers of η and then ϵ :

$$\psi_{0y}|_1+\epsilon[\cos\alpha(x-t)\psi_{0yy}|_1+\psi_{1y}|_1]+\epsilon^2[\frac{\psi_{0yyy}|_1}{2}\cos^2\alpha(x-t)+\psi_{2y}|_1+$$
$$\cos\alpha(x-t)\psi_{1yy}|_1]+\cdots=-\mathrm{Kn}\{\psi_{0yy}|_1+\epsilon[\cos\alpha(x-t)\psi_{0yyy}|_1+\psi_{1yy}|_1]+$$
$$\epsilon^2[\frac{\psi_{0yyyy}|_1}{2}\cos^2\alpha(x-t)+\cos\alpha(x-t)\psi_{1yyy}|_1+\psi_{2yy}|_1]+\cdots\}, \tag{6}$$

$$\psi_{0x}|_1+\epsilon[\cos\alpha(x-t)\psi_{0xy}|_1+\psi_{1x}|_1]+\epsilon^2[\frac{\psi_{0xyy}|_1}{2}\cos^2\alpha(x-t)+$$
$$\cos\alpha(x-t)\psi_{1xy}|_1+\psi_{2x}|_1]+\cdots=-\epsilon\alpha\sin\alpha(x-t). \tag{7}$$

Equations above, together with the condition of symmetry and a uniform $(\partial p/\partial x)_0$, yield :

$$\psi_0=K_0[(1+2\mathrm{Kn})y-\frac{y^3}{3}], \qquad K_0=\frac{Re}{2}(-\frac{\partial p}{\partial x})_0, \tag{8}$$

$$\psi_1=\frac{1}{2}\{\phi(y)e^{i\alpha(x-t)}+\phi^*(y)e^{-i\alpha(x-t)}\}, \tag{9}$$

where the asterisk denotes the complex conjugate. A substitution of ψ_1 into Eqn. (3) yields

$$\{\frac{d^2}{dy^2}-\alpha^2+i\alpha Re[1-K_0(1-y^2+2\mathrm{Kn})]\}(\frac{d^2}{dy^2}-\alpha^2)\phi-2i\alpha K_0 Re\,\phi=0$$

or if originally the (matter) gas is quiescent : $K_0=0$ (this corresponds to a free (vacuum) pumping case)

$$(\frac{d^2}{dy^2}-\alpha^2)(\frac{d^2}{dy^2}-\bar{\alpha}^2)\phi=0, \qquad \bar{\alpha}^2=\alpha^2-i\alpha Re. \tag{10}$$

The boundary conditions are

$$\phi_y(\pm 1)\pm\phi_{yy}(\pm 1)\mathrm{Kn}=2K_0(1\pm\mathrm{Kn})=0, \qquad \phi(\pm 1)=\pm 1. \tag{11}$$

Similarly, with

$$\psi_2=\frac{1}{2}\{D(y)+E(y)e^{i2\alpha(x-t)}+E^*(y)e^{-i2\alpha(x-t)}\}, \tag{12}$$

we have

$$D_{yyyy}=-\frac{i\alpha Re}{2}(\phi\phi^*_{yy}-\phi^*\phi_{yy})_y, \tag{13}$$

$$[\frac{d^2}{dy^2}-(4\alpha^2-2i\alpha Re)](\frac{d^2}{dy^2}-4\alpha^2)E-i2\alpha ReK_0(1-y^2+2\mathrm{Kn})$$

$$(\frac{d^2}{dy^2}-4\alpha^2)E+i4\alpha K_0 ReE+\frac{i\alpha Re}{2}(\phi_y\phi_{yy}-\phi\phi_{yyy})=0; \tag{14}$$

and the boundary conditions

$$D_y(\pm 1)+\frac{1}{2}[\phi_{yy}(\pm 1)+\phi^*_{yy}(\pm 1)]-2K_0=\mp\mathrm{Kn}\{\frac{1}{2}[\phi_{yyy}(\pm 1)+\phi^*_{yyy}(\pm 1)]+D_{yy}(\pm 1)\}, \tag{15}$$

$$E_y(\pm 1)+\frac{1}{2}\phi_{yy}(\pm 1)-\frac{K_0}{2}=\mp\mathrm{Kn}[\frac{1}{2}\phi_{yyy}(\pm 1)+E_{yy}(\pm 1)], \tag{16}$$

$$E(\pm 1)+\frac{1}{4}\phi_y(\pm 1)=0 \tag{17}$$

where K_0 is zero in Eqns. (13-16).

2.3 Originally Quiescent Gas

To simplify the approach and obtain preliminary analytical solutions of above complicated equations and boundary conditions, we only consider the case in which $(\partial p/\partial x)_0$ or $K_0=\psi_0=0$. After lengthy algebraic manipulations, we obtain

$$\phi=c_0e^{\alpha y}+c_1e^{-\alpha y}+c_2e^{\bar{\alpha}y}+c_3e^{-\bar{\alpha}y},$$

where $c_0=(A+A_0)/Det$, $c_1=-(B+B_0)/Det$, $c_2=(C+C_0)/Det$, $c_3=-(T+T_0)/Det$;

$$Det=Ae^{\alpha}-Be^{-\alpha}+Ce^{\bar{\alpha}}-Te^{-\bar{\alpha}},$$

$$A=e^{\alpha}\bar{\alpha}^2(r^2e^{-2\bar{\alpha}}-s^2e^{2\bar{\alpha}})-2\alpha\bar{\alpha}e^{-\alpha}w+\alpha\bar{\alpha}e^{\alpha}z(e^{-2\bar{\alpha}}r+e^{2\bar{\alpha}}s),$$

$$A_0=e^{-\alpha}\bar{\alpha}^2(r^2e^{-2\bar{\alpha}}-s^2e^{2\bar{\alpha}})+2\alpha\bar{\alpha}e^{\alpha}z-\alpha\bar{\alpha}e^{-\alpha}w(e^{2\bar{\alpha}}s+e^{-2\bar{\alpha}}r),$$

$$B=e^{-\alpha}\bar{\alpha}^2(r^2e^{-2\bar{\alpha}}-s^2e^{2\bar{\alpha}})+2\alpha\bar{\alpha}e^{\alpha}z-\alpha\bar{\alpha}e^{-\alpha}w(e^{-2\bar{\alpha}}r+e^{2\bar{\alpha}}s),$$

$$B_0=e^{\alpha}\bar{\alpha}^2(r^2e^{-2\bar{\alpha}}-s^2e^{2\bar{\alpha}})-2\alpha\bar{\alpha}e^{-\alpha}w+\alpha\bar{\alpha}e^{\alpha}z(e^{-2\bar{\alpha}}r+e^{2\bar{\alpha}}s),$$

$$C=e^{-\alpha}\alpha\bar{\alpha}(wse^{\bar{\alpha}-\alpha}-rze^{\alpha-\bar{\alpha}})-\alpha e^{2\alpha+\bar{\alpha}}z(\alpha z-\bar{\alpha}s)+\alpha e^{-\alpha}w(\alpha e^{\bar{\alpha}-\alpha}w-\bar{\alpha}e^{\alpha-\bar{\alpha}}r),$$

$$C_0=e^{\alpha}\alpha\bar{\alpha}(wse^{\bar{\alpha}-\alpha}-rze^{\alpha-\bar{\alpha}})-\alpha z(z\alpha e^{2\alpha-\bar{\alpha}}-\bar{\alpha}e^{\bar{\alpha}}s)+\alpha w(\alpha e^{-(\bar{\alpha}+2\alpha)}w-\bar{\alpha}e^{-(2\alpha+\bar{\alpha})}r),$$

$$T=e^{-\alpha}\alpha\bar{\alpha}(zse^{\bar{\alpha}+\alpha}-rwe^{-(\alpha+\bar{\alpha})})-\alpha\bar{\alpha}(e^{2\alpha-\bar{\alpha}}zr-e^{\bar{\alpha}}ws)+\alpha^2e^{-\alpha}(-e^{2\alpha}z^2+e^{-2\alpha}w^2),$$

$$T_0=e^{\alpha}\alpha\bar{\alpha}(zse^{\bar{\alpha}+\alpha}-rwe^{-(\alpha+\bar{\alpha})})-\alpha\bar{\alpha}(e^{-\bar{\alpha}}zr-e^{\bar{\alpha}-2\alpha}ws)+\alpha^2e^{\alpha}(-e^{2\alpha}z^2+e^{-2\alpha}w^2),$$

with $r=(1-\bar{\alpha}\mathrm{Kn})$, $s=(1+\bar{\alpha}\mathrm{Kn})$, $w=(1-\alpha\mathrm{Kn})$, $z=(1+\alpha\mathrm{Kn})$.
To obtain a simple solution which relates to the mean transport so long as only terms of $O(\epsilon^2)$ are concerned, we see that if every term in the x-momentum equation is averaged over an interval

of time equal to the period of oscillation (due to vacuum fluctuations), we obtain for our solution as given by above equations the time-averaged (unit) body forcing

$$\overline{\frac{\partial p}{\partial x}} = \epsilon^2 (\overline{\frac{\partial p}{\partial x}})_2 = \epsilon^2 [\frac{D_{yyy}}{2Re} + \frac{iRe}{4}(\phi\phi^*_{yy} - \phi^*\phi_{yy})] + O(\epsilon^3) = \epsilon^2 \frac{\Pi_0}{Re} + O(\epsilon^3), \tag{18}$$

where Π_0 is the integration constant for the integration of equation (12) and could be fixed indirectly in the coming equation (22). Now, from Eqn. (14), we have

$$D_y(\pm 1) \pm \mathrm{Kn} D_{yy}(\pm 1) = -\frac{1}{2}[\phi_{yy}(\pm 1) + \phi^*_{yy}(\pm 1)] \mp \mathrm{Kn}\{\frac{1}{2}[\phi_{yyy}(\pm 1) + \phi^*_{yyy}(\pm 1)]\}, \tag{19}$$

where $D_y(y) = \Pi_0 y^2 + a_1 y + a_2 + \mathcal{C}(y)$, and together from equation (12), we obtain

$$\begin{aligned} \mathcal{C}(y) = & \frac{\alpha^2 Re^2}{2}[\frac{c_0 c_2^*}{g_1^2} e^{(\alpha+\bar{\alpha}^*)y} + \frac{c_0^* c_2}{g_2^2} e^{(\alpha+\bar{\alpha})y} + \frac{c_0 c_3^*}{g_3^2} e^{(\alpha-\bar{\alpha}^*)y} + \frac{c_0^* c_3}{g_4^2} e^{(\alpha-\bar{\alpha})y} + \\ & \frac{c_1 c_2^*}{g_3^2} e^{(\bar{\alpha}^*-\alpha)y} + \frac{c_1^* c_2}{g_4^2} e^{(\bar{\alpha}-\alpha)y} + \frac{c_1 c_3^*}{g_1^2} e^{-(\bar{\alpha}^*+\alpha)y} + \frac{c_1^* c_3}{g_2^2} e^{-(\bar{\alpha}+\alpha)y} + \\ & \frac{c_2 c_3^*}{g_5^2} e^{(\bar{\alpha}-\bar{\alpha}^*)y} + \frac{c_2^* c_3}{g_5^2} e^{(\bar{\alpha}^*-\bar{\alpha})y} + 2\frac{c_2 c_2^*}{g_6^2} e^{(\bar{\alpha}^*+\bar{\alpha})y} + 2\frac{c_3 c_3^*}{g_6^2} e^{-(\bar{\alpha}^*+\bar{\alpha})y}], \end{aligned} \tag{20}$$

with $g_1 = \alpha + \bar{\alpha}^*$, $g_2 = \alpha + \bar{\alpha}$, $g_3 = \alpha - \bar{\alpha}^*$, $g_4 = \alpha - \bar{\alpha}$, $g_5 = \bar{\alpha} - \bar{\alpha}^*$, $g_6 = \bar{\alpha} + \bar{\alpha}^*$. In realistic applications we must determine Π_0 from considerations of conditions at the ends of the matter-region. a_1 equals to zero because of the symmetry of boundary conditions.
Once Π_0 is specified, our solution for the mean speed (u averaged over time) of matter-flow is

$$U = \epsilon^2 \frac{D_y}{2} = \frac{\epsilon^2}{2}\{\mathcal{C}(y) - \mathcal{C}(1) + R_0 - \mathrm{Kn}\,\mathcal{C}_y(1) + \Pi_0[y^2 - (1 + 2\mathrm{Kn})]\} \tag{21}$$

where $R_0 = -\{[\phi_{yy}(1) + \phi^*_{yy}(1)] - \mathrm{Kn}[\phi_{yyy}(1) + \phi^*_{yyy}(1)]\}/2$, which has a numerical value about 3 for a wide range of α and Re (playing the role of viscous dissipations) when Kn= 0. To illustrate our results clearly, we adopt $U(Y) \equiv u(y)$ for the time-averaged results with $y \equiv Y$ in the following.

3 Results and Discussion

We check our approach firstly by examining R_0 with that of no-slip (Kn= 0) approach. This can be done easily once we consider terms of $D_y(y)$ and $\mathcal{C}(y)$ because to evaluate R_0 we shall at most take into account the higher derivatives of $\phi(y)$, like $\phi_{yy}(y)$, $\phi_{yyy}(y)$ instead of $\phi_y(y)$ and escape from the prescribing of a_2.
Our numerical calculations confirm that the mean streamwise velocity distribution (averaged over time) due to the induced motion by the wavy elastic vacuum-matter interface in the case of free (vacuum) pumping is dominated by R_0 (or Kn) and the parabolic distribution $-\Pi_0(1-y^2)$. R_0 which defines the boundary value of D_y has its origin in the y-gradient of the first-order streamwise velocity distribution, as can be seen in Eqn. (14).
In addition to the terms mentioned above, there is a perturbation term which varies across the channel : $\mathcal{C}(y) - \mathcal{C}(1)$. Let us define it to be

$$F(y) = \frac{-200}{\alpha^2 Re^2}[\mathcal{C}(y) - \mathcal{C}(1)] \tag{22}$$

To compare with no-slip (Kn= 0) results, we plot three cases, $\alpha = 0.1, 0.4$, and 0.8 for the same Reynolds number $Re = 1$ of our results : Kn= 0.1 with those Kn= 0 into Fig. 2. We remind the readers that the Reynolds number here is based on the wave speed. This figure confirms our approaches since we can recover no-slip results by checking curves of Kn= 0 and finding them being almost completely matched in-between. The physical trend herein is also the same as those reported in Refs. [12-13] for the slip-flow effects. The slip produces decoupling with the inertia of the wavy interface.

Now, let us define a critical reflux condition as one for which the mean velocity $U(Y)$ equals to zero at the center-line $Y = 0$ (cf. Fig. 2). With equations (12,20-21), we have

$$\Pi_{0_{cr}} = Re(\overline{\frac{\partial p}{\partial x}})_2 = \frac{[\alpha^2 Re^2 F(0)/200 + \mathrm{Kn}\, \mathcal{C}'(1) - R_0]}{-(1+2\mathrm{Kn})} \tag{23}$$

which means the critical reflux condition is reached when Π_0 has above value. Pumping against a positive (unit) body forcing greater than the critical value would result in a backward transport (reflux) in the central region of the stream. This critical value depends on α, Re, and Kn. There will be no reflux if the (unit) body forcing or pressure gradient is smaller than this Π_0. Thus, for some Π_0 values less than $\Pi_{0_{cr}}$, the matter (flow) will keep moving or evolving forward. On the contrary, parts of the matter (flow) will move or evolve backward if $\Pi_0 > \Pi_{0_{cr}}$. This result could be similar to that in [16] using different approach or qualitatively related to that of [5] : even for very slow growth of Λ, the gravitationally bound systems become unbound while the nongravitationally bound systems remain bound for certain parameters defined in [5] (e.g., η).

3.1 Possible Massless and Negative-Pressure States

We present some of the values of $\Pi_0(\alpha, Re; \mathrm{Kn} = 0, 0.1)$ corresponding ot freezed or zero-volume-flow-rate states ($\int_{-1}^{1} U(Y)dY = 0$) in Table 1 where the wave number (α) has the range between 0.20 and 0.80; the Reynolds number (Re)= $0.1, 1, 10, 100$. We observe that as Kn increases from zero to 0.1, the critical Π_0 or time-averaged (unit) body forcing decreases significantly. For the same Kn, once Re is larger than 10, critical reflux values Π_0 drop rapidly and the wave-modulation effect (due to α) appears. The latter observation might be interpreted as the strong coupling between the vacuum-matter interface and the inertia of the streaming matter-flow. The illustration of the velocity fields for those zero-flux (zero-volume-flow-rate) or freezed states or massless states are shown in Figure 3. There are three wave numbers : $\alpha = 0.2, 0.5, 0.8$. The Reynolds number is 10. Both no-slip and slip (Kn=0.1) cases are presented. The arrows for slip cases are schematic and represent the direction of positive and negative velocity fields.

Some remarks could be made about these states : the matter or universe being freezed in the time-averaged sense for specific dissipations (in terms of Reynolds number which is the ratio of wave-inertia and viscous dissipation effects) and wave numbers (due to the wavy vacuum-interface or vacuum fluctuations) for either no-slip and slip cases. This particular result might also be related to a changing cosmological term (growing or decaying slowly) or the critical density mentioned in Refs. [1-2]. If we treat the (unit) body forcing as the pressure gradient, then for the same transport direction (say, positive x-direction), the negative pressure (either

downsdtream or upstream) will, at least, occur once the time-averaged flow (the maximum speed of the matter (gas) appears at the center-line) is moving forward! For example, for $\Pi_0 = Re(\overline{\frac{\partial p}{\partial x})_2} = -10 < 0$ ($Re = 1, \alpha = 0.5$), the velocity field (profile) is shown in Fig. 4. One possible p-pair for uniform (negative) gradient (mean value theorem): $p_{downstream} - p_{upstream} < 0$, with $x_{downstream} - x_{upstream} > 0 : p_{downstream} < 0$, $p_{downstream} < p_{upstream} < 0$.

The possible explanation for this latter result (negative-pressure states) is : Please be reminded that our results are time-averaged (averaged over an interval of time equal to the period of oscillation (due to vacuum fluctuations)) and are already in steady state or the 2nd.-order transport (even though the primary state of the matter gas is quiescent without the Big-Bang explosion-driven force) is almost fully developed (which means the viscous dissipation being balanced by the wave inertia coming from the elastic interface or vacuum fluctuations). The quasi-stationary state after taking time-average is a balanced combination of both energy-input and energy-dissipation with the former coming from the small-amplitude elastic deformations of the entire configuration (elastic energy) and the latter coming from the incomplete exchange of the momenta or energy along the vacuum-matter interface and the internal friction of the matter (Navier-Stokes gases with viscosities prescribed). The present approach is not exactly the same as previous approaches (say, [5,9]). There are indeed perturbed 2nd.-order time-averaged effects and parts of them could induce the negative-pressure states demonstared above (cf. Fig. 4) for specific Reynolds numbers, wave numbers, and Knudsen numbers (one is illustrated above)!

Under the same situation, we further demonstrate the effects of viscous dissipation in Figs. 5 ($Re = 100, \alpha = 0.2, \Pi_0 = -50$) and 6 ($Re = 0.1, \alpha = 0.2, \Pi_0 = -0.1$). Once d and c (the wave speed) are fixed, the former corresponds to the case of less viscous dissipation while the latter corresponds to the case of much more viscous dissioation. Meanwhile, the time-averaged transport induced by the wavy interface is proportional to the square of the amplitude ratio (although the small amplitude waves being presumed), as can be seen in Eqn. (12) or (20), which is qualitatively the same as that presented in [9] for analogous interfacial problems. In brief summary, using the boundary perturbation methods, the entrained transport or pattern : either postive or negative and there is possibility : freezing (which could be linked to the dissipative evolution of the Universe) due to the wavy vacuum-matter interface is mainly tuned by the (unit) body forcing or Π_0 for fixed Re. Meanwhile, $\Pi_{0_{cr}}$ depends strongly on the Knudsen number (Kn, a rarefaction measure) instead of Re or α. We also noticed that it has been recently pointed out that fluctuations in the stress-energy tensor of quantum fields may be important for some states in curved spacetimes or even flat spacetimes with nontrivial topology [16]. Hu and Phillips, for instance, computed the energy density fluctuations of quantum states in spatially closed Friedmann-Robertson-Walker models and showed that these could be as important as the energy density itself [17]. If so, these fluctuations may induce relevant backreaction effects on the gravitational field (the spacetime geometry). Some of our results shown above resemble these latter results. We hope that in the future we can investigate other issues like the role of phase transition and that of cyclic universes [18-21] using the present or more advanced approach.

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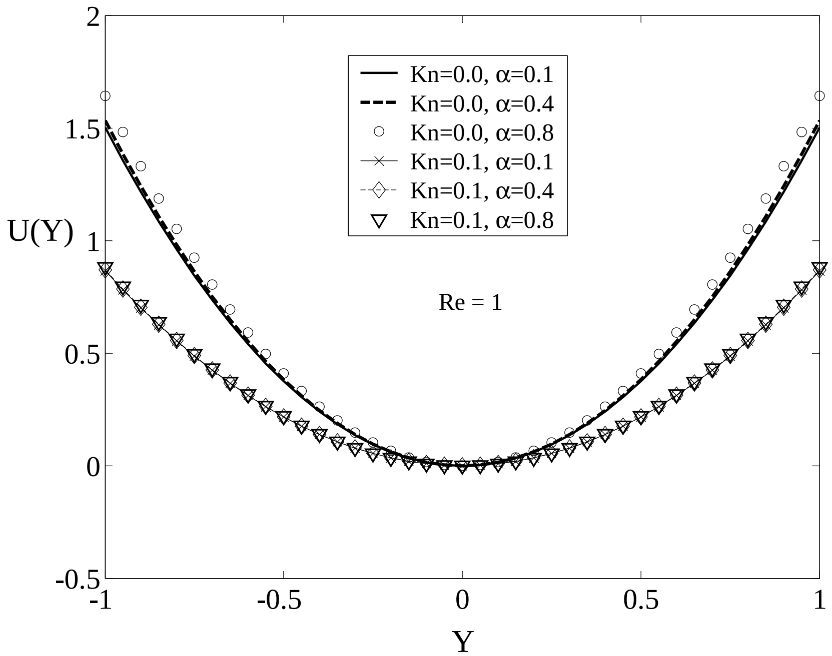

Fig. 2 Demonstration of Kn and α effects on the time-averaged velocity (fields) profiles. $\Pi_0 = \Pi_{0cr}$. The Reynolds number (the ratio of the wave-inertia and viscous dissipation effects) is 1. $U(Y) = 0$ at $Y = 0$. Kn is the Knudsen number (a rarefaction measure).

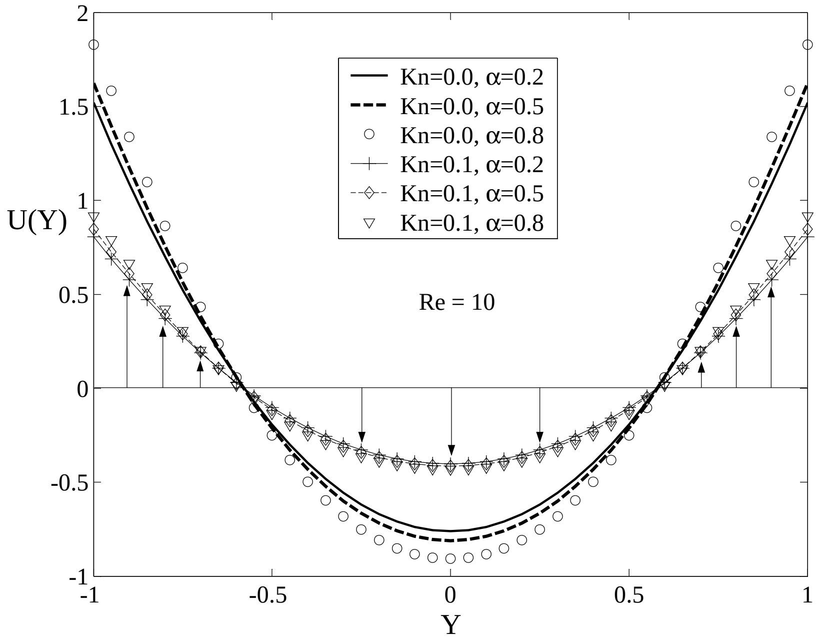

Fig. 3 Demonstration of the zero-flux states : the mean velocity field $U(Y)$ for wave numbers $\alpha = 0.2, 0.5, 0.8$. The Reynolds number is 10. Kn is the rarefaction measure (the mean free path of the particles divided by the characteristic length).
The arrows are schematic and illustrate the directions of positive and negative $U(Y)$.
The integration of $U(Y)$ w.r.t. Y for these velocity fields gives zero volume flow rate.

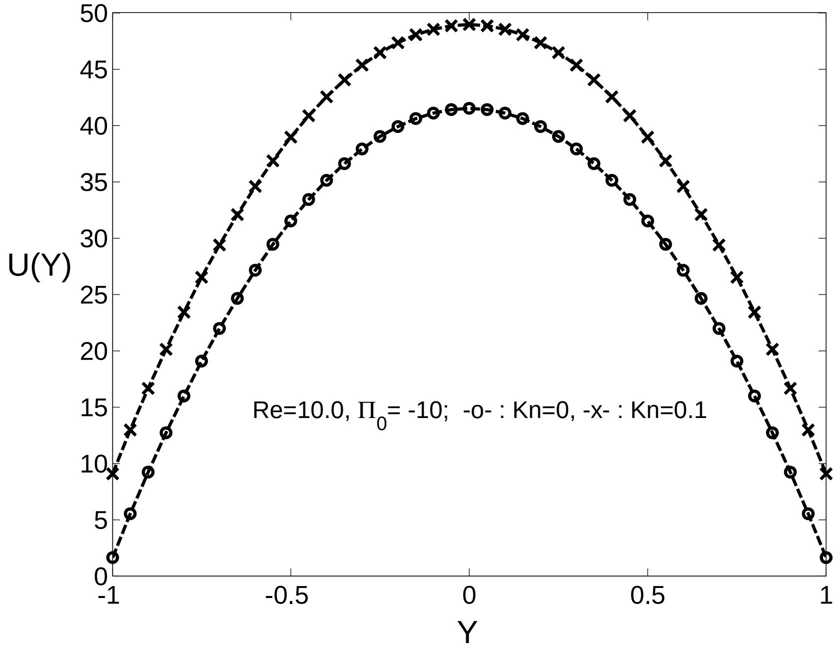

Fig. 4 Demonstration of the negative-p states : the mean velocity field $U(Y)$ for the wave number $\alpha = 0.2$ and the Reynolds number $Re = 10$.

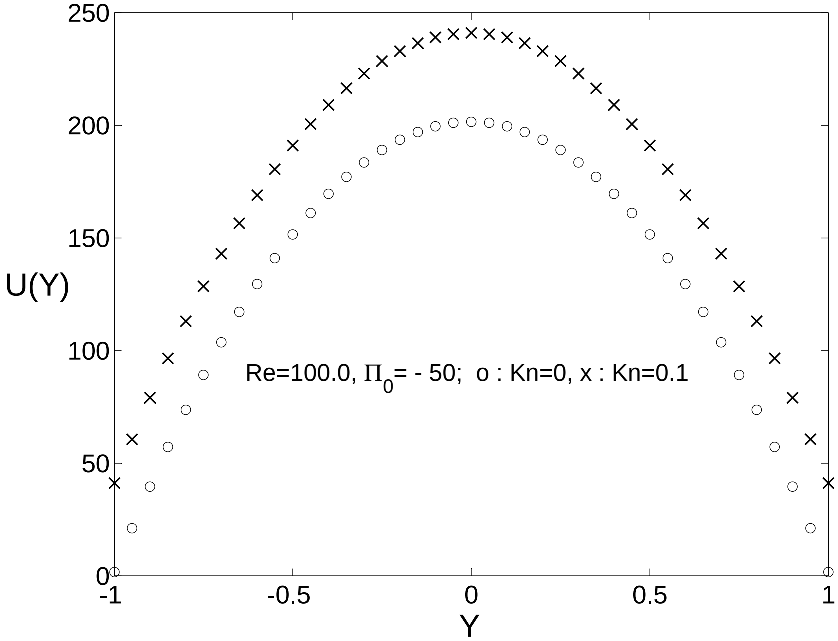

Fig. 5 Demonstration of the negative-p states : the mean velocity field $U(Y)$ for the wave number $\alpha = 0.2$ and the Reynolds number $Re = 100$. $\Pi_0 = -50$. As $Re = c\ d/\nu$, once c (the wave speed) is fixed, this transport corresponds to the case with less viscous dissipation.

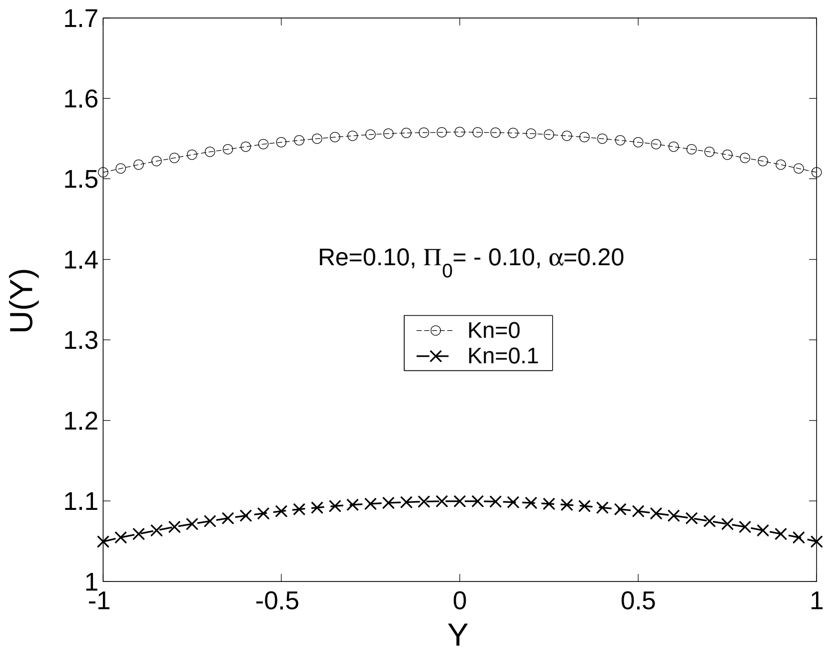

Fig. 6 Demonstration of the negative-p states : the mean velocity field $U(Y)$ for the wave number $\alpha = 0.2$ and the Reynolds number $Re = 0.1$. $\Pi_0 = -0.1$. As $Re = c\ d/\nu$, once c (the wave speed) is fixed, this transport corresponds to the case with much more viscous dissipation.

Table 1: Zero-flux or freezed states values (Π_0) for a flat vacuum-matter interface.

		Re			
Kn	α	0.1	1	10	100
0	0.2	4.5269	4.5269	4.5231	4.3275
	0.5	4.6586	4.6584	4.6359	4.2682
	0.8	4.9238	4.9234	4.8708	4.4488
0.1	0.2	2.4003	2.4000	2.3774	1.2217
	0.5	2.4149	2.4132	2.2731	-0.9054
	0.8	2.4422	2.4379	2.0718	-3.4151